

$$
\begin{gathered}
A=\pi r^{2} \\
a^{2}+b^{2}=c^{2} \\
y=m x+b
\end{gathered}
$$

## Curriculum Guide

## For Developmental

## Mathematics

## Units 1 - 9

With Basic Skills Unit on Whole Numbers

Presented by the Developmental Math Curriculum Team February 2011

Updated July 2011

## Dear Fellow Mathematics Faculty Member:

We are pleased to present this Curriculum Guide. It represents the culmination of our work during fall 2010. As a team of 25 mathematics faculty members representing all 23 colleges in the VCCS together with System Office support, we began meeting last September to develop this Curriculum Guide. While the Curriculum Guide was in development, our drafts were widely shared and vetted with mathematics faculty. Each college received one vote at the table when decisions were made.

The August 2010 report of the Developmental Mathematics Redesign Team, The Critical Point: Redesigning Developmental Mathematics Education in Virginia's Community Colleges, set the stage for our work. We focused on:

1. Developing student learning outcomes for each unit
2. Developing curricular materials, to include:
a. Suggested timelines for each topic
b. Sample assessments
c. Teaching tips
d. Sample syllabi
3. Determining what constitutes mastery
4. Keeping lines of communication open to our fellow faculty members on campus
5. Assisting system office staff in examining implications for the Student Information System (SIS), financial aid, and student success research
6. Addressing other implementation issues identified in the report such as the feasibility of offering a college-level mathematics course such as MTH 158 in place of the advanced algebra units as well as using alternate grading schemes

The guiding principle for our work was that the goal of developmental mathematics is to prepare students for college-level mathematics, for other courses dependent on a mathematical curriculum foundation, and for general education purposes. "The existence of a course or program must be based on more than 'this is material that students should have had a high school'. This philosophy assumes that high schools and colleges have the same educational mission." (From Developmental Mathematics Symposium at AMATYC 2009, Karr and Rotman)

It will be up to each individual college to determine the delivery method for the material and the textbook to be used. The student learning outcomes have been developed to clearly state the minimum material that must be covered in the unit. This is what any student transferring to another college and having completed a unit would be expected to know. Within each individual unit, you do not have to teach the material in the order described, but all topics must be covered. The teaching tips were driven by discussion within the colleges and among the curriculum team members to better clarify the depth and breadth of the topics listed in the student learning outcomes.

This document is intended to be dynamic and to be revised periodically for continuous improvement. Your suggestions for improvements are welcome and will be accepted on an ongoing basis. Please submit your suggestions for teaching tips, interesting contextual or application problems, and other ideas to deved@vccs.edu. Please visit www.vccs.edu/deved to see what is happening in developmental education in the VCCS.

We look forward to working with you to meet the aggressive student success goal adopted by the VCCS to improve student progression from developmental mathematics, through college level mathematics, and through graduation or transfer.

Sincerely,

## The Curriculum Team for Developmental Mathematics

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## Introduction

This Curriculum Guide is designed to provide information to colleges and faculty in developmental mathematics as they develop their instruction and assessments for the various units in developmental mathematics.

## Overview of Units

The content of the developmental mathematics curriculum reflects what is needed to be successful in college mathematics and college curricula. The goals are:

- To cover selected topics thoroughly and allow students to master skills before moving to the next topic
- To streamline the developmental experience and to eliminate those topics that are taught within precalculus and calculus (e.g. synthetic division)
- To enable students to take or repeat only those sections with which they need help
- To enable students to complete only those units that are required as prerequisites for the curriculum specific college-level course required in the student's chosen program of study


## Basic Skills

The developmental mathematics entry-level unit is operation with positive fractions. Mastery of operations and basic concepts with whole numbers is presumed for unit las outlined in Appendix A. Options for students who need to gain skills in whole number operations, include offering BSK 1, the basic skills course in whole numbers, or providing independent study tutorials such as online software, Key Train or Win. The college should provide appropriate resources on campus to assist students in preparing for enrollment in Unit 1. Such resources may include:

- Coordination with local ABE programs to deliver necessary content
- Bridge workshops or mini-courses delivered through Non-credit programs
- On-line lessons created using learning software (e.g., MyMathTest or ALEKS) with tutor support in the college math lab


## Developmental Mathematics Units

Critical thinking and application problems will be infused throughout the units. Each unit is one credit. One Academic Hour is 50 minutes and sixteen Academic Hours is equal to one credit. Developmental credits do not apply to any certificate or degree. Each unit assumes competency in all previous units. The skills from previous units should be reinforced in subsequent units. Geometry concepts should be included whenever possible in each of the units.

| Unit | Unit Name | Unit Description |
| :---: | :---: | :---: |
| Unit 1 | Operations with Positive Fractions | The student will solve application problems using proper fractions, improper fractions, and mixed numbers. All student learning outcomes for this unit must be completed without the use of a calculator. Emphasis should be placed on applications throughout the unit. Applications will use U.S. customary units of measurement. All fractions in this unit should be expressed in simplest form, unless otherwise indicated. |
| Unit 2 | Operations with Positive Decimals and Percents | The student will solve problems using decimals and percents. Emphasis should be placed on applications throughout the unit. Applications will use U.S. customary and metric units of measurement. |
| Unit 3 | Algebra basics | The student will perform basic operations with algebraic expressions and solve simple algebraic equations using signed numbers. Emphasis should be placed on applications throughout the unit. |
| Unit 4 | First Degree Equations and Inequalities in one variable | The student will solve first degree equations and inequalities containing one variable, and use them to solve application problems. Emphasis should be on learning the steps to solving the equations and inequalities, applications and problem solving. |
| Unit 5 | Linear Equations, Inequalities and Systems of Linear equations in Two Variables | The student will learn how to find the equation of a line, graph linear equations and inequalities in two variables and solve a system of two linear equations. Emphasis should be on writing and graphing equations using the slope of the line and points on the line, and applications. |
| Unit 6 | Exponents, Factoring and Polynomial Equations | The student will learn techniques to factor polynomials and use these techniques to solve polynomial equations. Emphasis should be on learning all the different factoring methods, and solving application problems using polynomial equations. |
| Unit 7 | Rational <br> Expressions and Equations | The student will simplify rational algebraic expressions, solve rational algebraic equations and use them to solve application problems. |
| Unit 8 | Rational <br> Exponents and Radicals | The student will simplify radical expressions, and use rational exponents. The student will solve radical equations and use them to solve application problems. |
| Unit 9 | Functions, <br> Quadratic <br> Equations and Parabolas | In this unit the student will have an introduction to functions in ordered pair, graph, and equation form. The student will engage in a thorough introduction to quadratic functions and their properties as they complete preparation for entering STEM or business-administration college-level mathematics courses |

## Course Syllabus

Each course syllabus must be consistent with the course student learning outcomes as described in the Curriculum Guide. See Appendix B for some sample syllabi of the units.

Within each individual unit, the order in which the topics are listed does not necessarily imply that topics must be covered in that order.

## Questions about the Units

## Which units should a student take?

The required pre-requisite units for college-level mathematics courses are in the appropriate course descriptions in the master course file.

How will a student be placed in a unit?
A Placement and Diagnostic system will be developed to determine the units needed.
How do students progress through the units?
Students must demonstrate mastery of units with scores of at least $75 \%$ on the final assessment of each unit before taking the subsequent unit or course.

What grades will students receive in the units?
Currently, the grading for developmental courses is S=Satisfactory, $\mathrm{R}=$ Reenroll, U= Unsatisfactory, and $\mathrm{W}=$ Withdraw.

## How many times can a student take the final assessment for a unit?

The Curriculum Team recommends that the student may have no more than three attempts to pass the final assessment before having to repeat the unit.

How many times can a student repeat a unit?
In keeping with VCCS policy, only two attempts are allowed before student must seek special permission to repeat.

## Can a student take units out of sequence?

Units are sequential. The pre-requisite for a unit is successful completion of the previous unit or a qualifying score on the placement instrument.

What are the recommendations regarding the use of a calculator?
Unit 1 is to be completed without the use of a calculator. The decision to use a calculator in Units $2-9$ is left up to each individual college.

## Unit 1 - Operations with Positive Fractions

## Unit Description

The student will solve application problems using proper fractions, improper fractions, and mixed numbers. All student learning outcomes for this unit must be completed without the use of a calculator. Emphasis should be placed on applications throughout the unit. Applications will use U.S. customary units of measurement. All fractions in this unit should be expressed in simplest form, unless otherwise indicated.

## Broad Learning Outcomes

Upon completion of Unit 1 students will be able to:
1.1 Write, simplify, and compare fractions.
1.2 Perform operations with fractions.
1.3 Solve applications using U.S. customary units of measurement.

## Specific Objectives

Upon completion of Unit 1 students will be able to:
1.1 Write, simply and compare fractions.
1.1.1 Express parts of a whole using fraction notation.
1.1.2 Convert between improper fractions and mixed numbers.
1.1.3 Express repeated factors using exponents.
1.1.4 Find the prime factorization of a given number.
1.1.5 Write fractions in simplest form.
1.1.6 Compare two quantities in the form of a ratio or rate in simplest form.
1.1.7 Find the least common multiple (LCM) of two or more whole numbers.
1.1.8 Find the least common denominator (LCD) of two or more fractions.
1.1.9 Determine the relationship (<, >, =) between two fractions with unlike denominators.
1.2 Perform operations with fractions.
1.2.1 Add and subtract fractions and mixed numbers with like
denominators.
1.2.2 Add and subtract fractions and mixed numbers with unlike denominators.
1.2.3 Multiply fractions and mixed numbers.
1.2.4 Divide fractions and mixed numbers.
1.2.5 Simplify expressions involving fractions using order of operation.
1.3 Solve application using U.S. customary units of measurement.

## Suggested Time

7 academic hours

6 academic hours

2 academic hours

## Sample Assessment for Unit 1

1.1.1
1.1.2
1.1.4

1. If you have four quarters, three dimes, two nickels, and two pennies, what fraction of the whole coin collection is in dimes?
2. Write $5 \frac{7}{8}$ as an improper fraction.
3. Write $\frac{29}{5}$ as a mixed number.
4. Express $3 \bullet 3 \bullet 3 \bullet 5 \bullet 5$ using exponents.
5. Find the prime factorization of 180. Express your answer in exponential form.
6. Write $\frac{24}{90}$ in simplest form.
7. Write the ratio of 14 days to 3 weeks in simplest form.
8. Write the ratio of 4 teachers to 134 students in simplest form.
9. Write the rate of 400 miles in 8 hours in simplest form.
10. Find the LCM for 8,12 , and 36 .
11. Find the LCD for $\frac{1}{5}, \frac{1}{4}$, and $\frac{1}{9}$.
12. Enter the correct symbol $(<,>,=)$. $\frac{3}{4} \quad \frac{11}{15}$
13. Add or subtract. $\frac{3}{16}+\frac{5}{16}$
14. Add or subtract. $\frac{25}{64}-\frac{17}{64}$
15. Add or subtract. $4 \frac{1}{4}-2 \frac{3}{4}$
16. Add or subtract. $\frac{3}{12}+\frac{4}{50}$
17. Add or subtract. $\frac{7}{18}-\frac{5}{24}$
18. Add or subtract. $41-23 \frac{3}{7}$
1.2.3
1.2.3
1.2.4
1.2.4
19. Multiply. $\frac{12}{56} \bullet \frac{14}{42}$
20. Multiply. $1 \frac{5}{16} \bullet \frac{3}{4}$
21. Divide. $\frac{\frac{4}{15}}{\frac{5}{6}}$
22. Divide. $4 \frac{3}{7} \div 1 \frac{2}{9}$
23. Simplify the expression. $\frac{3}{4}-\frac{2}{5} \bullet \frac{7}{12}$
24. Simplify the expression. $\frac{3}{4}+\frac{5}{9} \bullet \frac{12}{15}$
25. If 15 of the students are male and 18 of the students are female in a math class, what fractional part of the class is female?
26. The depth of tire tread on a new tire is $\frac{9}{32}$ inch. After two months use, $\frac{1}{16}$ inch has been worn off. What is the depth of the remaining tire tread?
27. If you lose $3 \frac{1}{2}$ pounds the first week of your diet and $2 \frac{2}{3}$ pounds the second week, how many pounds do you still need to lose to reach your goal of losing 10 pounds?
28. If you are paid $\$ 5.50 /$ hour for mowing yards, and you take $3 \frac{1}{3}$ hours to mow a yard, how much money are you owed?
29. If a full oil barrel holds $53 \frac{3}{4}$ gallons of oil, and $10 \frac{3}{4}$ gallons are used every week, how many weeks will the oil last?
30. Your new iPod Shuffle holds 500 songs. You have loaded 310 of your favorite tunes onto your iPod. Represent the number of songs on your iPod as a fraction of the total of number of songs it will hold. Simplify if possible.
31. A surgery patient needs to lose 24 pounds before her upcoming surgery. Your patient lost $6^{1 / 2}$ pounds at the first 2 week checkup and $82 / 5$ pounds at the next visit. How much more does the patient need to lose to achieve the weight lose needed for surgery?
32. You are purchasing the ingredients for your first cooking project in culinary school. You are making a new salmon dish. Salmon costs $\$ 6$ per pound. Find the cost of $3 \frac{2}{3}$ pounds.
33. You have budgeted $2 / 5$ of your monthly income for rent and utilities. Your monthly income is $\$ 2100$.
a) What amount have you budgeted for rent and utilities?
b) What amount is left over for other expenditures during the month
34. Your car uses $15 \frac{1}{2}$ gallons of gasoline on a 310 mile trip. Find your car's miles per gallon.
35. The nurse's orders are to give the child $3 \frac{1}{2}$ tablespoons of medicine per day in 4 equally divided doses. How much medicine is to be given at each dose?
36. To maintain financial aid, a student must pass at least $\frac{2}{3}$ of their attempted credits each semester. If you are taking 16 credits this semester, how many credits must you pass?

## Teaching Tips for Unit 1

1. When at all possible, use measuring devices when working with application problems. For example, you can copy "rulers" on paper that can be used to measure objects in the classroom. You can also use thermometers, scales and weights.
2. Exponential notation is introduced in 1.2.1 for the purpose of writing prime factorizations in exponential form. In 1.2.1, be sure students understand that $2 \cdot 2 \cdot 2=2^{3}$ so that the prime factorization of 72 can be written as $2^{3} \cdot 3^{2}$.

## Unit 2 - Operations with Positive Decimals and Percents

## Unit Description

The student will solve problems using decimals and percents. Emphasis should be placed on applications throughout the unit. Applications will use U.S. customary and metric units of measurement.

## Broad Learning Outcomes

Upon completion of Unit 2 students will be able to:
2.1 Demonstrate the meaning of decimal numbers.
2.2 Perform operations with decimals.
2.3 Estimate decimals.
2.4 Demonstrate the relationship among fractions, decimals, and percents.
2.5 Solve basic percent problems.
2.6 Read and interpret basic graphs.
2.7 Convert units of measure.
2.8 Solve application problems using U.S. customary and metric units of measurement.

2.6 Read and interpret basic graphs.2.6.1 Read and interpret information from a pie graph.
2.6.2 Calculate the percentage denoted by a pie graph.
2.6.3 Read and interpret information from a bar graph.
2.6.4 Read and interpret information from a line graph.
2.7 Convert units of measure.
2.7.1 Convert within the U.S. system.
2.7.2 Convert within the metric system.
2.7.3 Convert between U.S. and metric units using conversion tables.
2.7.4 Convert units of time.
2.7.5 Convert between Fahrenheit and Celsius temperatures.
2.8 Solve application problems using U.S. customary and metric units of
measurement.

1 academic hour

## Sample Assessment for Unit 2

2.1.1 1. Write "twenty-one and fifty-six thousandths" as a number.
2.1.2 $\quad$ 2. Identify the place value of the digit 7 in the number 0.4715 .
2.2.1 3. Add. $9.69+1.4$
2.2.2 4. Multiply 0.35 and 2.9.
2.2.3 5. Divide 52.7 by 3.4.
2.2.4 6. Simplify. $2.4+3.15 \cdot 2$
2.3.1 7. Round 0.4327 to the nearest hundredth.
2.3.2 8. Estimate $8.24+6.4705$ by first rounding each number to the nearest whole number. Give your estimate as a whole number.
2.3.2 9. Estimate $3.49 \times 7.508$ by first rounding each number to the nearest whole number. Give your estimate as a whole number.
2.4.1 10. The square below contains 100 smaller squares, and 25 of them are shaded. What percent of the whole is shaded?

11. Write $5 / 8$ as a percent.
12. Convert $3 / 16$ to a decimal.
13. Convert 0.83 to a fraction
14. Write $20 \%$ as a fraction in simplest form.
15. Write $12.5 \%$ as a decimal.
16. Order the list of fractions and decimals from smallest to largest. $5 / 6,0.658,7 / 10,0.75$
17. 30 is what percent of 150 ?
18. Mrs. Toner's fifth-grade class increased from 16 students last year to 20 students this year. What percent increase is this?
2.5.2 19. At the local paper mill, the number of industrial accidents decreased from 15 accidents last month to just 9 accidents this month. Find the percent decrease.
2.5.3 20. A laptop computer is priced at $\$ 650$. If the sales tax is $5 \%$, find the total cost of the computer, including sales tax.
23. In the pie chart below, what percent of the Johnson's weekly expenses are spent on clothing?

## Johnson Family Weekly Expenses


24. The pie chart below shows sales by quarter for Smith Plumbing. If the total sales for the year are $\$ 360,000$, calculate the sales for the $3^{\text {rd }}$ Quarter.

## Yearly Sales for Smith Plumbing


25. Using the bar graph below, determine how many Juniors are taking a math class if there are a total of 450 Juniors at the high school.

## Students Taking a Math Class


2.7.1
2.7.1
2.7.2
2.7.2
2.7.2
2.7.3
2.7.3
2.7.4
27. Convert 7 feet to inches.
28. Convert 24 ounces to pounds.
29. Convert 1.28 meters to centimeters.
30. Convert 2.6 kilograms to grams.
31. Convert 750 milliliters to liters.
32. Using the conversion table, convert 5 quarts to liters.
33. Using the conversion table, convert 10 centimeters to inches.
34. Convert $2 \frac{1}{2}$ hours to minutes.
35. Convert 68 degrees Fahrenheit to Celsius, using the formula $C=\frac{5}{9}(F-32)$.
36. Convert 15 degrees Celsius to Fahrenheit, using the formula $F=\frac{9}{5} C+32$.
37. Jason is serving a 10 -kilogram turkey to 28 people. How many grams of meat is he allowing for each person? Round to the nearest whole gram.
38. Dan orders supplies for the science labs. Each of the 24 stations in the chemistry lab needs 2 feet of rubber tubing. If rubber tubing sells for $\$ 8.75$ per yard, how much will it cost to equip all the stations?

## Teaching Tips for Unit 2

1. When at all possible, use measuring devices when working with application problems. For example, you can copy "rulers" on paper that can be used to measure objects in the classroom. You can also use thermometers, beakers, scales and weights.
2. Use play money to reinforce concepts with decimals.
3. Provide conversion charts for converting with units.

## Unit 3 - Algebra Basics

## Unit Description

The student will perform basic operations with algebraic expressions and solve simple algebraic equations using signed numbers. Emphasis should be placed on applications throughout the unit.

## Broad Learning Outcomes

Upon completion of Unit 3 students will be able to:
3.1 Determine the absolute value of a number.
3.2 Demonstrate proper use of exponents.
3.3 Find the principal square root of a perfect square.
3.4 Simplify expressions involving signed numbers.
$3.5 \quad$ Write numbers in scientific notation.
3.6 Simplify algebraic expressions.
3.7 Evaluate a formula or algebraic expression for given values of the variables.
3.8 Solve one-step equations using the addition and multiplication properties.
3.9 Solve problems using proportions.
3.10 Solve application problems including finding perimeter, area and volume.

## Specific Objectives

Upon completion of Unit 3 students will be able to:
Suggested Time
$1 / 2$ academic hour
$1 / 2$ academic hour
3.2.1 Express repeated factors using exponents.
3.2.2 Evaluate powers of numbers.
3.3 Find the principal square root of a perfect square.
3.4 Simplify expressions involving signed numbers.
3.4.1 Add and subtract signed numbers.
3.4.2 Multiply and divide signed numbers.
3.4.3 Use the proper order of operations to simplify expressions containing multiple operations on signed numbers, including powers and square roots.
3.5 Write numbers in scientific notation.

### 3.5.1 Convert between integer powers of 10 and equivalent decimal numbers.

3.5.2 Convert numbers between scientific notation and standard
notation.
3.6 Simplify algebraic expressions.
3.6.1 Identify the properties of real numbers (Commutative, Associative, Distributive, Identity and Inverse Properties).
3.6.2 Simplify an algebraic expression by combining like terms.
3.6.3 Simplify algebraic expressions using the order of operations.
3.7 Evaluate a formula or algebraic expression for given values of the variables.

2 academic hours

### 3.8 Solve one-step equations using the addition and multiplication properties. <br> 3.8.1 Solve one-step equations using rational numbers. <br> 3.8.2 Solve one-step equations using percents. <br> 3.9 Solve problems using proportions. <br> 3.10 Solve application problems including finding perimeter, area and $21 / 2$ academic hours <br> 1 academic hour <br> 1 academic hour <br> -

## Sample Assessment for Unit 3

3.1
3.1
3.2.1
3.2.1
3.2.2
3.2.2
3.3
3.4.1
3.4.1
3.4.2
3.4.2
3.4.3
3.4.3
3.4.3
3.4.3
3.5.1
3.5.2
3.5.2
3.6.1
3.6.2
3.6.3
3.7
3.7
3.8.1

1. Find the absolute value: $|-3|$
2. Find the absolute value: $|18|$
3. Write in exponential form: $13 \cdot 13 \cdot 13$
4. Write in exponential form: $1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$
5. Evaluate: $2^{3}$
6. Evaluate: $4^{2}$
7. Evaluate: $\sqrt{16}$
8. Perform the indicated operations and simplify: $6+(-10)$
9. Perform the indicated operations and simplify: $\frac{1}{8}-\left(-\frac{4}{3}\right)$
10. Perform the indicated operations and simplify: $-7(-3.1)$
11. Perform the indicated operations and simplify: $-72 \div(-9)$
12. Perform the indicated operations and simplify: $-4^{2}+6$
13. Perform the indicated operations and simplify: $-5+(-10)-(-4)-13$
14. Perform the indicated operations and simplify: $-32-8 \div 4-(-2)$
15. Perform the indicated operations and simplify: $2+\sqrt{4}(10-2)+3^{2}$
16. Write as a decimal number: $10^{-3}$
17. Write in scientific notation: $2,061,000,000$
18. Write in standard notation: $9.3 \times 10^{-2}$
19. Identify the property of real numbers that is being illustrated
a. $3 n+5=5+3 n$
b. $2 x+(y+z)=(2 x+y)+z$
c. $a(b+c)=a b+a c$
d. $b+-b=0$
e. $a+0=a$
f. $\mathrm{a} \cdot 1=\mathrm{a}$
g. $a \cdot \frac{1}{a}=1$
20. Combine like terms: $19 n+30 b-9 b+4 n$
21. Simplify completely: $5+3(\mathrm{x}-1)$
22. Evaluate when $x$ is $-5: x^{2}+2 x-1$
23. In the formula $A=\frac{1}{2} h(B+b)$, find $A$ when $h=10, B=20$, and $b=16$.
24. Solve: $n+7=-16$
3.8.1
3.8.1
3.8.1
3.8.2
25. Solve: $-\frac{2}{3}+x=-\frac{1}{6}$
26. Solve: $-8 \mathrm{x}=-72$
27. Solve: $-2.4+\mathrm{t}=5.6$
28. To obtain her bachelor's degree in nursing, Judy must complete 130 credit hours of instruction. If she has completed $60 \%$ of her requirement, how many credits did Judy complete?
29. A factory manufacturing low voltage relays found 4 defective relays in a lot of 80 relays. At this rate, how many defective relays can be expected in a lot of 740 relays?
30. The population of Lewisburg was 10,820 . It decreased by 320 each year for 5 consecutive years. What was the population after 5 years?
31. The baggage compartment of a bus is a rectangle prism. The dimensions of the baggage compartment are 8 ft by 4 ft by 6 ft . What is the volume of the compartment? What is the perimeter of the floor? What is the area of the floor?

8 ft
Volume
Perimeter of floor 4 ft Area of floor $\qquad$


## Teaching Tips for Unit 3

1. Introduce signed numbers at the beginning of the unit. Ensure that students can use a number line to represent integer values.
2. Ensure students can identify the different sets of numbers in this unit (whole, natural, integer, rational, irrational, real)
3. Include geometry applications whenever possible.
4. Perimeter and area problems should include triangles, rectangles, circles, squares, parallelograms and trapezoids.

## Unit 4 - First Degree Equations and Inequalities in One Variable

## Unit Description

The student will solve first degree equations and inequalities containing one variable, and use them to solve application problems. Emphasis should be on learning the steps to solving the equations and inequalities, applications and problem solving.

## Broad Learning Outcomes

Upon completion of Unit 4 students will be able to:
4.1 Solve first degree equations in one variable.
4.2 Solve a formula or equation for one of its variables.
4.3 Solve first degree absolute value equations containing a single absolute value.
4.4 Solve first degree inequalities in one variable.
4.5 Solve application problems using a single first degree equation or inequality.

## Specific Objectives

Upon completion of Unit 4 students will be able to:
4.1 Solve first degree equations in one variable.
4.1.1 $\quad$ Solve first degree equations in one variable using the Addition Property of Equality.
4.1.2 Solve first degree equations in one variable using the Multiplication Property of Equality.
$\begin{array}{ll}\text { 4.1.3 } & \text { Solve first degree equations in one variable using the Addition } \\ \text { Property of Equality and the Multiplication Property of Equality. }\end{array}$
4.1.4 Solve first degree equations in one variable that contain parentheses.
4.1.5 Solve first degree equations in one variable with the variable on both sides of the equal sign.
4.1.6
Solve first degree equations in one variable and identify the solution
to an equation as finite, the empty set or all real numbers.
4.2 Solve a formula or equation for one of its variables.
4.2.1 Solve a formula or equation for one of its variables using the Addition Property of Equality.
4.2.2 Solve a formula or equation for one of its variables using the Multiplication Property of Equality.
4.2.3 Solve a formula or equation for one of its variables using the Addition Property of Equality and the Multiplication Property of Equality.
4.3 Solve first degree absolute value equations containing a single absolute value.
4.4 Solve first degree inequalities in one variable.
4.4.1 Solve first degree inequalities in one variable stating the solution using inequality notation.
4.4.2 Solve first degree inequalities in one variable stating the solution using interval notation.
4.4.3 Solve first degree inequalities in one variable and graph the solution
on a real number line.
4.5 $\begin{aligned} & \text { Solve application problems using a single first degree equation or } \\ & \text { inequality. }\end{aligned}$

Suggested time
5 academic hours

2 academic hours

1 academic hour
2 academic hours

5 academic hours

## Sample Assessment for Unit 4

4.1.1
4.1.2
4.1.3
4.1.3
4.1.4
4.1.5
4.1.5
4.1.6
4.2.1
4.2.2
4.2.3

1. Solve the equation $x-2=9$.
2. Solve the equation $5 x=40$.
3. Solve the equation $5 x+7=42$ for $x$.
4. Solve the equation $\frac{x}{2.26}+8.31=-8.43$ for $x$. Round your answer to two decimal places.
5. Solve the equation $-3(y+6)=9$ for $y$.
6. Solve the equation $5+7 t=23+t$ for $t$.
7. Solve the equation $3(x+2)=-9(x-7)$ for $x$.
8. Solve the equation $-7(y+4)=-5 y-2 y-3$ and identify the solution as finite, the empty set, or all real numbers.
9. Solve the equation $S=A-D$ for A.
10. Solve the equation $D=r t$ for t .
11. Solve the equation $P=2 l+2 w$ for w .
12. Solve $|15-5 t|=55$, if possible.
13. Solve $2.5 x+5 \leq 16.4$. State the solution using inequality notation.
14. Solve the inequality $5(x+2) \geq 3(x+9)$. State the solution using inequality notation.
15. Solve the inequality $-\frac{3}{8} t \leq 5$. State the solution using interval notation.
16. Write the solution using interval notation.

17. Solve the inequality $x+3 \leq-2$. Graph the solution on a real number line.
18. A company pays its sales representatives 35 cents per mile if they use their personal cars. A sales representative submitted a bill to be reimbursed for $\$ 148.05$ for driving. How many miles did the sales representative drive?
19. If an object traveled 230 miles at a rate of 25 miles per hour, how long (in hours) did it take to travel this distance?
20. A hardware store is having a $20 \%$-off sale. If an item has a list price of $\$ 14.40$, what is the item's sale price?
21. Tickets for a college baseball game are $\$ 9$ for lower level seats and $\$ 5$ for upper level seats. For a particular game, 850 lower level seats were sold. The total revenue from the ticket sales was $\$ 9650$. How many upper level seats were sold?
22. The bill for repairing a car was $\$ 345$. The cost for parts was $\$ 160$. The cost for labor was $\$ 37$ per hour. How many hours did the repair work take?

## Teaching Tips for Unit 4

1. Encourage students to write all of the addition principle steps and to avoid using shortcuts until they have mastered these types of equations.
2. Encourage students to write the steps for solving the equations in a neat and organized manner. This habit will help immensely when the equations become more complex.
3. Encourage students to check their solutions.
4. Some students confuse the principles and try to subtract the coefficient from the variable instead of multiplying to obtain a coefficient of 1 .
5. Sometimes it is required for students to solve a number of problems using the same formula. It may be advantageous for students to rewrite the formula so that it is solved for the required letter first.
6. Encourage students to label answers with the correct units.
7. Encourage students to check whether their final answers are reasonable.
8. Some students are unfamiliar with < and > and need to be taught the definitions of each.
9. Review the common English-to-inequality translations.
10. Some students benefit from seeing solutions on the number line.

# Unit 5 - Linear Equations, Inequalities and Systems of Linear Equations in Two Variables 

## Unit Description

The student will learn how to find the equation of a line, graph linear equations and inequalities in two variables and solve a system of two linear equations. Emphasis should be on writing and graphing equations using the slope of the line and points on the line, and applications.

## Broad Learning Outcomes

Upon completion of Unit 5 students will be able to:
5.1 Define the properties of the rectangular coordinate system.
5.2 Graph a linear equation in two variables.
5.3 Graph a linear inequality in two variables.
5.4 Find the slope of a line.
5.5 Write an equation of a line.
5.6 Solve systems of linear equations.
5.7 Use function notation.
5.8 Solve application problems that require linear equations, inequalities and systems of linear equations in two variables.

## Specific Objectives

Upon completion of Unit 5 students will be able to:
Suggested time
5.1 Define the properties of the rectangular coordinate system.
5.1.1 Determine the coordinates of a point plotted on the coordinate
plane.
5.1.2 Determine whether an ordered pair is a solution to an equation in two variables.
5.1.3 Graph a linear equation by finding and plotting ordered pair solutions.
5.2 Graph a linear equation in two variables.
5.2.1 Identify the $x$ and $y$ intercepts of a graph.
5.2.2 Graph a linear equation by plotting intercepts.
5.2.3 Graph an equation given in slope-intercept form.
5.2.4 Graph a horizontal line given its equation.
5.2.5 Graph a vertical line given its equation.
5.3 Graph a linear inequality in two variables.
5.4 Find the slope of a line.
5.4.1 Find the slope of a line given two points on the line.
5.4.2 Find the slope of a line given its equation in slope-intercept form.
5.4.3 Find the slope of a line given its equation by converting to slopeintercept form.
5.4.4 Find the slope of a line given its graph.
5.4.5 Find the slope of horizontal and vertical lines.

2 academic hours

1 academic hour
2 academic hours

### 5.5 Write an equation of a line.

5.5.1 Write an equation of a line in slope-intercept form given the slope and the $y$-intercept.
5.5.2 Use point-slope form to write an equation of a line in slope intercept form given the slope and a point on the line.
5.5.3 Use point-slope form to write an equation of a line in slope intercept form given two points on the line.
5.5.4 Write the equation of a vertical line.
5.5.5 Write the equation of a horizontal line.
5.5.6 Find the equation of a line that is parallel or perpendicular to a given line and passes through a given point.
5.6 Solve systems of linear equations.
5.6.1 Determine if an ordered pair is a solution of system of equations in two variables.
5.6.2 Solve systems of linear equations by graphing.
5.6.3 Solve by elimination using substitution.
5.6.4 Solve by elimination using addition.
5.6.5 Identify a system of linear equations as consistent and independent, consistent and dependent, or inconsistent.
5.7 Use function notation.
5.7.1 Evaluate $\mathrm{y}=\mathrm{f}(\mathrm{x})$ for specific values of x .
5.7.2 Given the graph of $y=f(x)$, evaluate $f(x)$ for specific values of x .
5.7.3 Given the graph of $y=f(x)$, find $x$ for specific values of $f(x)$.
5.8 Solve applications problems that require linear equations, inequalities and systems of linear equations in two variables.
$2^{1 / 2}$ academic hours
$2^{1 / 2}$ academic hours

1 academic hour

2 academic hours

## Sample Assessment for Unit 5

5.1.1

1. Determine the coordinates of the point labeled 7

2. Determine which of the ordered pairs is a solution of the equation.

$$
x+5 y+10=0
$$

a) $(-3,4)$
b) $(0,-2)$
c) $(-5,10)$
d) $(4,-5)$
3. Graph the linear equation by completing the table.

4. Evaluate $f(x)=2 x-1$ at $x=6$.
5. Estimate the $x$ - and $y$-intercepts of the graph.

6. Graph the linear equation by plotting the $x$ - and $y$-intercepts. $-2 x+4 y-16=0$.
7. Sketch the graph of the equation $y=\frac{2}{3} x+2$ and label three points on the graph.
8. Graph. $y=4$
9. Graph. $x=-3$
10. Sketch the graph of the linear inequality.

$$
3 x-2 y \geq 6
$$

11. Find the slope (if possible) of the line passing through the points $(4,7)$ and $(5,10)$.
12. Find the slope of the graph of the equation $y=-\frac{1}{6} x+1$.
13. Find the slope of the equation by converting to slope-intercept form.

$$
5 x+8 y=-7
$$

14. Find the slope of the given line.

15. Find the slope of $x=-8$, if possible
16. Find the slope of $y=3$, if possible
17. Write the equation of the line that passes through the point $\left(0,-\frac{4}{9}\right)$ and has slope $m=-\frac{9}{7}$. Write the equation in slope-intercept form.
18. Write the equation of the line that passes through the point $(5,-5)$ and has slope $m=-5$. Write the equation in slope-intercept form.
19. Write an equation of the line that passes through the points $(-1,-10)$ and $(-5,2)$. Write the equation in slope-intercept form.
20. Write an equation of a line passing through the points $(-5,7)$ and $(-5,2)$.
21. Write an equation of a line passing through the points $(1,2)$ and $(14,2)$.
22. Write an equation of the line through the point $(4,-2)$ that is parallel to the line $-5 x+6 y=-2$
23. Write an equation of the line through the point $(5,3)$ that is perpendicular to the line $-3 x+7 y=5$.
24. Determine whether $(-4,6),(7,-4),(-4,5),(-1,-1)$, or $(5,8)$ is a solution of the system of equations below.
$\left\{\begin{array}{l}4 x+7 y=0 \\ 2 x+9 y=-22\end{array}\right.$
25. Sketch the graphs of the equations $\left\{\begin{array}{l}x-y=2 \\ x+y=2\end{array}\right.$ and approximate any solutions of the system of linear equations.
26. If possible, solve the system below by elimination using substitution.

$$
\left\{\begin{array}{r}
2 x-3 y=-9 \\
x+y=-2
\end{array}\right.
$$

27. If possible, solve the system below by elimination using addition.
$\left\{\begin{aligned} a-2 b & =7 \\ 4 a+2 b & =8\end{aligned}\right.$
28. Identify the system of linear equations as consistent and independent, consistent and dependent, or inconsistent.
$\left\{\begin{aligned} 4 x-4 y & =4 \\ 2 x-y & =6\end{aligned}\right.$
29. Given $f(x)=3 x-6$ evaluate each of the following: $f(5), f(-2), f\left(\frac{2}{3}\right), f(0), f(6.2)$
30. Given the graph of $y=f(x)$ below, evaluate $f(-6), f(0), f(-2), f(1)$

31. Given the graph of $y=f(x)$ below, find $x$ such that $f(x)=4$

32. A hot-air balloon at 1020 feet descends at a rate of 85 feet per minute. Let $y$ represent the height of the balloon and let $x$ represent the number of minutes the balloon descends. Write an equation that relates the height of the hot-air balloon to the number of minutes it descends.
33. You invest a total of $\$ 5800$ in two investments earning 3.5\% and $5.5 \%$ simple interest. Your goal is to have a total annual interest income of $\$ 283$. Write a system of linear equations that represents this situation where $x$ represents the amount invested in the $3.5 \%$ fund and $y$ represents the amount invested in the $5.5 \%$ fund. Solve this system to determine the smallest amount that you can invest at $5.5 \%$ in order to meet your objective.

## Teaching Tips for Unit 5

1. Some students are unfamiliar with graphing and need to see the number line graph first, and then a perpendicular $y$-axis added to it to make a rectangular coordinate system.
2. Some students are very confused by the fact that they can choose any value of $x$ or $y$ as a starting point for finding an ordered-pair solution.
3. Many students have trouble creating the correct graphing scale on applied problems.
4. Many students do not understand the equations $\mathrm{x}=\mathrm{c}$ and $\mathrm{y}=\mathrm{c}$ and must memorize which case gives a horizontal line and which case gives a vertical line.
5. Students sometimes mix-up the $x$ - and $y$-intercepts.
6. Most students need to hear a qualitative description of slope before using the slope formula. For example, show sketches of lines with,,+- 0 , and undefined slope and discuss what happens to $y$ as $x$ increases. Then the slope formula gives a quantitative measure of the change in $y$ as $x$ increases.
7. Many students make sign errors when calculating the slope.
8. Use function notation whenever possible.

## Unit 6 - Exponents, Factoring and Polynomial Equations

## Unit Description

The student will learn to perform operations on exponential expressions and polynomials. Students will also learn techniques to factor polynomials and use these techniques to solve polynomial equations. Emphasis should be on learning all the different factoring methods, and solving application problems using polynomial equations.

## Broad Learning Outcomes

Upon completion of Unit 6 students will be able to:
6.1 Perform operations on exponential expressions using the rules of exponents.
6.2 Define, add, subtract, multiply and divide polynomials.
6.3 Factor polynomials.
6.4 Solve polynomial equations using factoring techniques.
6.5 Solve application problems involving polynomial equations and factoring.

## Specific Objectives

Upon completion of Unit 6 students will be able to:
6.1 Perform operations on exponential expressions using the rules of exponents.
6.1.1 Evaluate the product of two exponential expressions.
6.1.2 Evaluate the quotient of two exponential expressions.
6.1.3 Evaluate the power of a power of an exponential expression.
6.1.4 Evaluate exponential expressions that contain negative exponents.
6.1.5 Evaluate exponential expressions that contain combinations of products, quotients, power of a power and negative exponents.
6.1.6 Multiply and divide numbers in Scientific Notation.
6.2 Define, add, subtract, multiply and divide polynomials.
6.2.1 Identify an expression as a monomial, binomial, trinomial or polynomial.
6.2.2 Add, subtract, multiply and divide monomials using the rules of exponents.
6.2.3 Add, subtract, and multiply binomials.
6.2.4 Add, subtract, and multiply trinomials.
6.2.5 Add, subtract, and multiply combinations of binomials and trinomials.
6.3 Factor polynomials.
6.3.1 Find the greatest common factor from a list of terms.
6.3.2 Find the greatest common factor from a polynomial.
6.3.3 Factor a polynomial by grouping.
6.3.4 Factor trinomials of the form $x^{2}+b x+c$.
6.3.5 Factor trinomials of the form $a x^{2}+b x+c, a \neq 1$.
6.3.6 Factor a difference of squares.
6.3.7 Factor a sum of two cubes.
6.3.8 Factor a difference of two cubes.

### 6.4 Solve polynomial equations using factoring techniques. <br> 6.5 Solve application problems involving polynomial equations and

2 academic hours 2 academic hours

## Sample Assessment for Unit 6

6.1.1
6.1.2
6.1.3
6.1.4
6.1.5
6.1.5
6.1.5
6.1.6
6.2.1
6.2.2
6.2.2
6.2.4
6.2.3
6.2.4

1. Simplify the expression $\left(-7 x^{8}\right) x^{3}$.
2. Simplify the expression $\frac{5^{6} u^{6} v^{8}}{5^{8} u^{6} v^{4}}$. Assume that neither $u$ nor $v$ is zero.
3. Simplify the expression $\left(s^{6} t\right)^{2}$.
4. Find the value of $(-2)^{-6}$.
5. Rewrite the expression $\frac{5 w^{3}}{4 z^{-3}}$ using only positive exponents. Assume that $z$ is not zero.
6. Use the rules of exponents to simplify the expression $\left(2 x^{9} y^{8}\right)^{-4}$ using only positive exponents. Assume that neither $x$ nor $y$ is 0 .
7. Rewrite the expression using only positive exponents, and simplify. Assume that any variables in the expression are nonzero.

$$
\frac{\left(2 a^{-2} b^{4}\right)^{4} b}{\left(6 a^{2} b\right)^{3}}
$$

8. Perform the indicated operations. Write each answer in scientific notation and then without exponents.
a) $\frac{9.5 \times 10^{4}}{5.3 \times 10^{-3}}$
b) $\left(9.3 \times 10^{-5}\right)\left(6 \times 10^{3}\right)$
9. Is the polynomial below a monomial, binomial, trinomial, or none of these?

$$
8 x^{3}+4 x
$$

10. Find the sum $17 x+5 x$
11. Multiply $\left(-5 y^{3}\right)\left(-3 y^{4}\right)$.
12. Perform the division.

$$
\frac{10 z^{3}+3 z^{2}-2 z}{2 z}
$$

13. Find the sum $(9 x+4)+(6 x-6)$.
14. Find the difference $(-5 x+4)-(-3 x+7)$.
15. Multiply $(5 x+6)(9 x+7)$ and simplify.
16. Find the difference $\left(5 x^{2}-2 x-3\right)-\left(3 x^{3}-9 x^{2}-3\right)$.
6.2 .5
6.2.5
6.2.5
6.3.1
6.3.2
6.3.3
6.3.4
6.3.4
6.3.5
6.3.5
6.3.6
6.3.7
6.3.8

Perform the indicated operations and simplify.
17.

$$
\left(-5 x^{4}+8 x^{2}+2\right)+7\left(5 x^{4}-3 x^{2}\right)
$$

18. Find the difference $\left(-7 x^{2}-7 x-7\right)-\left[\left(7 x^{3}-9 x^{2}-20\right)+(4 x-11)\right]$.
19. Multiply $\left(x^{2}+4 x-3\right)\left(5 x^{2}+7\right)$ and simplify.
20. Find the greatest common factor of $9 g^{8} h^{7}, 3 g^{9} h^{8}$, and $3 g^{2} h^{8}$,
21. Factor the polynomial $3 x y+15 x^{2} y-21 x^{6} y^{6}$.
22. Factor the polynomial by grouping, if possible. $4 y^{3}+3 y^{2}+28 y+21$
23. Factor the trinomial, if possible. $z^{2}+14 z+48$
24. Factor the trinomial, if possible. $m^{2}-7 m-120$
25. Factor the given polynomial $3 z^{2}+30 z+27$ completely.
26. Factor the trinomial, if possible. $20 g^{2}-63 g-21$
27. Factor the difference of two squares: $25 x^{2}-81 y^{2}$
28. Factor the polynomial, if possible. $27 m^{3}+64 n^{3}$
29. Factor the polynomial below, if possible. $z^{3}-125$
30. Solve the equation $2 x^{2}=76 x$ by factoring.
31. Solve the equation below by factoring. $a^{2}+8 a+29=13$
32. Solve the equation below by factoring. $x^{3}-17 x^{2}+72 x=0$
33. An object is thrown upward from a height of 192 feet with an initial velocity of 64 feet per second. The height $h$ (in feet) of the object after $t$ seconds is modeled by the equation $h=-16 t^{2}+64 t+192$.

How long will it take for the object to reach the ground?
The revenue $R$ from the sale of $x$ cameras is given by $R=55 x-x^{2}$. The cost $C$ of and sold in order to break even?

## Teaching Tips for Unit 6

1. Some students like to set up polynomials vertically to add or subtract, aligning the like terms.
2. Some students forget to subtract every term when aligning vertically.
3. Encourage students to memorize the formulas for multiplying a sum and a difference of two terms and also a square of a binomial. These will come in handy for factoring later.
4. Some students have trouble remembering the formulas. Encourage them to think of multiplying polynomials as distributing. As long as each term in one polynomial is distributed onto each term in the other polynomial, they will get the right answer.
5. When squaring a binomial, many students square both terms rather than multiply the binomial by itself.
6. Some students need to rewrite the coefficients in a problem in factored form in order to identify the common factors.
7. Encourage students to check their factorization by multiplying.
8. In some factoring problems students find it helpful to make a table listing all possible factor pairs for $c$ in the first column and their sums in the second column.
9. Many students find it difficult at first to factor trinomials where the constant term is negative.
10. Remind students that when the constant term is positive, the factor pairs they pick must have the same sign, while if the constant term is negative, the factor pairs must have opposite signs.
11. Many students are confused at first by a leading coefficient that is negative. Remind them to factor out a negative as a common factor.
12. Remind students to always look first for a common factor.
13. Many students need a quick review of factoring out a common binomial before attempting the factoring by grouping.
14. Many students are amazed that two different grouping approaches can lead to the same final answer. Encourage them to try different groupings so that they will start to see patterns of grouping arrangements that make the factoring easier.
15. Some students understand the difference of squares formula, $A^{2}-B^{2}=(A+B)(A-B)$, better if problems are first done using trinomial factoring (with a $0 x$ middle term).
16. Encourage students to become proficient with special case factoring as it will be important for future algebra topics such as completing the square.
17. Encourage students to always check if the first and last terms of a trinomial are perfect squares. If they are, the perfect square trinomial factoring might apply.
18. Encourage students to be organized in their approach to factoring and to keep track of products they have tried.
19. When solving quadratic equations remind students to always get zero on one side before factoring.
20. When solving applications encourage students to make a diagram whenever possible.
21. Many students find the applied problems difficult and need to see many examples.
22. Remind students to check whether their answers are reasonable for applied problems.
23. Use function notation whenever possible.
24. Include geometry concepts in the application problems whenever possible.

## Unit 7: Rational Expressions and Equations

## Unit Description

The student will simplify rational algebraic expressions, solve rational algebraic equations and use them to solve application problems.

## Broad Learning Outcomes

Upon completion of Unit 7 students will be able to:
7.1 Identify a rational algebraic expression.
7.2 Simplify rational algebraic expressions.
7.3 Perform arithmetic operations with rational algebraic expressions.
7.4 Solve rational algebraic equations.
7.5 Solve application problems using rational algebraic equations.

## Specific Objectives

Upon completion of Unit 7 students will be able to:
7.1 Identify a rational algebraic expression.
$\begin{array}{ll}\text { 7.1.1 } & \begin{array}{l}\text { Identify the real value of the variable for which a rational } \\ \text { algebraic expression having a denominator of the form } \mathrm{ax}+\mathrm{b} \\ \text { is undefined. }\end{array} \\ \text { 7.1.2 } & \begin{array}{l}\text { Identify all real values of the variable for which a rational } \\ \text { algebraic expression having a denominator of the form } \\ \text { ax }^{2}+\mathrm{bx}+\mathrm{c} \text { is undefined. }\end{array} \\ \text { 7.1.3 } & \begin{array}{l}\text { Express a rational algebraic expression having negative } \\ \text { exponents as an equivalent rational expression without negative } \\ \text { exponents. }\end{array}\end{array}$
$\begin{array}{ll}\text { 7.1.1 } & \begin{array}{l}\text { Identify the real value of the variable for which a rational } \\ \text { algebraic expression having a denominator of the form } \mathrm{ax}+\mathrm{b} \\ \text { is undefined. }\end{array} \\ \text { 7.1.2 } & \begin{array}{l}\text { Identify all real values of the variable for which a rational } \\ \text { algebraic expression having a denominator of the form } \\ \text { ax }^{2}+\mathrm{bx}+\mathrm{c} \text { is undefined. }\end{array} \\ \text { 7.1.3 } & \begin{array}{l}\text { Express a rational algebraic expression having negative } \\ \text { exponents as an equivalent rational expression without negative } \\ \text { exponents. }\end{array}\end{array}$
$\begin{array}{ll}\text { 7.1.1 } & \begin{array}{l}\text { Identify the real value of the variable for which a rational } \\ \text { algebraic expression having a denominator of the form } \mathrm{ax}+\mathrm{b} \\ \text { is undefined. }\end{array} \\ \text { 7.1.2 } & \begin{array}{l}\text { Identify all real values of the variable for which a rational } \\ \text { algebraic expression having a denominator of the form } \\ \text { ax }^{2}+\mathrm{bx}+\mathrm{c} \text { is undefined. }\end{array} \\ \text { 7.1.3 } & \begin{array}{l}\text { Express a rational algebraic expression having negative } \\ \text { exponents as an equivalent rational expression without negative } \\ \text { exponents. }\end{array}\end{array}$
$\begin{array}{ll}\text { 7.1.1 } & \begin{array}{l}\text { Identify the real value of the variable for which a rational } \\ \text { algebraic expression having a denominator of the form } \mathrm{ax}+\mathrm{b} \\ \text { is undefined. }\end{array} \\ \text { 7.1.2 } & \begin{array}{l}\text { Identify all real values of the variable for which a rational } \\ \text { algebraic expression having a denominator of the form } \\ \text { ax }^{2}+\mathrm{bx}+\mathrm{c} \text { is undefined. }\end{array} \\ \text { 7.1.3 } & \begin{array}{l}\text { Express a rational algebraic expression having negative } \\ \text { exponents as an equivalent rational expression without negative } \\ \text { exponents. }\end{array}\end{array}$
$\begin{array}{ll}\text { 7.1.1 } & \begin{array}{l}\text { Identify the real value of the variable for which a rational } \\ \text { algebraic expression having a denominator of the form } \mathrm{ax}+\mathrm{b} \\ \text { is undefined. }\end{array} \\ \text { 7.1.2 } & \begin{array}{l}\text { Identify all real values of the variable for which a rational } \\ \text { algebraic expression having a denominator of the form } \\ \text { ax }^{2}+\mathrm{bx}+\mathrm{c} \text { is undefined. }\end{array} \\ \text { 7.1.3 } & \begin{array}{l}\text { Express a rational algebraic expression having negative } \\ \text { exponents as an equivalent rational expression without negative } \\ \text { exponents. }\end{array}\end{array}$
$\begin{array}{ll}\text { 7.1.1 } & \begin{array}{l}\text { Identify the real value of the variable for which a rational } \\ \text { algebraic expression having a denominator of the form } \mathrm{ax}+\mathrm{b} \\ \text { is undefined. }\end{array} \\ \text { 7.1.2 } & \begin{array}{l}\text { Identify all real values of the variable for which a rational } \\ \text { algebraic expression having a denominator of the form } \\ \text { ax }^{2}+\mathrm{bx}+\mathrm{c} \text { is undefined. }\end{array} \\ \text { 7.1.3 } & \begin{array}{l}\text { Express a rational algebraic expression having negative } \\ \text { exponents as an equivalent rational expression without negative } \\ \text { exponents. }\end{array}\end{array}$
$\begin{array}{ll}\text { 7.1.1 } & \begin{array}{l}\text { Identify the real value of the variable for which a rational } \\ \text { algebraic expression having a denominator of the form } \mathrm{ax}+\mathrm{b} \\ \text { is undefined. }\end{array} \\ \text { 7.1.2 } & \begin{array}{l}\text { Identify all real values of the variable for which a rational } \\ \text { algebraic expression having a denominator of the form } \\ \text { ax }^{2}+\mathrm{bx}+\mathrm{c} \text { is undefined. }\end{array} \\ \text { 7.1.3 } & \begin{array}{l}\text { Express a rational algebraic expression having negative } \\ \text { exponents as an equivalent rational expression without negative } \\ \text { exponents. }\end{array}\end{array}$
$\begin{array}{ll}\text { 7.1.1 } & \begin{array}{l}\text { Identify the real value of the variable for which a rational } \\ \text { algebraic expression having a denominator of the form } \mathrm{ax}+\mathrm{b} \\ \text { is undefined. }\end{array} \\ \text { 7.1.2 } & \begin{array}{l}\text { Identify all real values of the variable for which a rational } \\ \text { algebraic expression having a denominator of the form } \\ \text { ax }^{2}+\mathrm{bx}+\mathrm{c} \text { is undefined. }\end{array} \\ \text { 7.1.3 } & \begin{array}{l}\text { Express a rational algebraic expression having negative } \\ \text { exponents as an equivalent rational expression without negative } \\ \text { exponents. }\end{array}\end{array}$
$\begin{array}{ll}\text { 7.1.1 } & \begin{array}{l}\text { Identify the real value of the variable for which a rational } \\ \text { algebraic expression having a denominator of the form } \mathrm{ax}+\mathrm{b} \\ \text { is undefined. }\end{array} \\ \text { 7.1.2 } & \begin{array}{l}\text { Identify all real values of the variable for which a rational } \\ \text { algebraic expression having a denominator of the form } \\ \text { ax }^{2}+\mathrm{bx}+\mathrm{c} \text { is undefined. }\end{array} \\ \text { 7.1.3 } & \begin{array}{l}\text { Express a rational algebraic expression having negative } \\ \text { exponents as an equivalent rational expression without negative } \\ \text { exponents. }\end{array}\end{array}$
7.2 Simplify rational algebraic expressions.
7.2.1 Simplify a rational algebraic expression.
7.2.2 Evaluate a rational algebraic expression given specific integral values for each variable.
7.3 Perform arithmetic operations with rational algebraic expressions.
Suggested time
1 academic hour
7.3.1 Perform addition and subtraction of rational algebraic expressions having like denominators.
7.3.2 Find the Least Common Denominator (LCD) of two or more rational algebraic expressions.
7.3.3 Perform addition and subtraction of rational algebraic expressions having denominators that have no common factors.
7.3.4 Perform addition and subtraction of rational algebraic expressions having denominators that have a common monomial factor.
7.3.5 Perform addition and subtraction of rational algebraic expressions having denominators that have a common binomial factor.
7.3.6 Perform multiplication of rational algebraic expressions and express the product in simplest terms. facto.

3 academic hours

8 academic hours

8 acal
7.3.7 Use factorization to divide rational algebraic expressions and express the quotient in simplest terms.
7.3.8 Simplify complex fractions.
7.3.9 Divide a polynomial by a monomial.
7.3.10 Perform polynomial long division having binomial divisors of the form $a x+b$.

### 7.4 Solve rational algebraic equations.

7.5 Solve application problems using rational algebraic equations.
7.5.1 Write a rational equation to match the information given in an application problem.
7.5.2 Solve an application problem using rational equations.

1 academic hour
2 academic hours

## Sample Assessment for Unit 7

7.1.1

1. Identify all the values of $x$ what make this expression undefined. $\frac{x-3}{4 x-7}$
2. Identify all the values of x what make this expression undefined. $\frac{v-2}{v^{2}-7 v+12}$
3. Rewrite this expression without negative exponents. $a^{-4} b^{2} c^{-2}$
4. Simplify. $\frac{2 x-6}{4 x-12}$
5. Given $p=2$ and $=-3$,
evaluate. $\frac{p^{2}-t^{2}}{4 p+4 t}$
6. Perform the indicated operations and simplify.
$\frac{w}{w^{2}-6 w-7}+\frac{-7}{w^{2}-6 w-7}$
7. Identify the Least Common Denominator of the two rational expressions.

$$
\frac{4}{x-2} \text { and } \frac{6}{x+2}
$$

8. Perform the indicated operations and simplify.
$\frac{4}{x-2}-\frac{6}{x+2}$
9. Perform the indicated operations and simplify.

$$
\frac{1}{7 y}+\frac{2}{9 y^{2}}
$$

7.3.5
7.3.6
7.3.7
10. Perform the indicated operations and simplify.
$\frac{m+7}{m^{2}+7 m+6}+\frac{2 m+1}{m^{2}+5 m+4}$
11. Perform the indicated operations and simplify.
$\frac{x^{2}-x}{x^{2}-6 x+8} \cdot \frac{x-4}{x^{2}+4 x} \div \frac{4 x}{x^{2}-6 x+8}$
12. Perform the indicated operations and simplify.

$$
\frac{8}{4-5 p}-\frac{2}{5 p-4}+\frac{p-4}{5 p^{2}+16 p-16}
$$

13. Simplify.

14. Simplify.

$$
\frac{\frac{w}{w^{2}-6 w-7}+\frac{-7}{w^{2}-6 w-7}}{\frac{w}{w^{2}-2 w-3}-\frac{4}{w^{2}-2 w-3}}
$$

15. Divide:
$\frac{18 j^{4}+45 j^{2}+27 j}{3 j}$
16. Use long division to find the quotient and express the remainder (if any) as a fraction. $\left(20 p^{3}+17 p^{2}+26 p+50\right) \div(4 p+5)$
17. Solve. $\frac{4}{x-2}=1+\frac{6}{x+2}$
18. The current in the Red Cedar River is 6 mph . A canoe can travel 7 miles downstream in the same time that it takes to travel 3 miles upstream when paddled at the same rate. Set up (but do not solve) a rational equation that could be used to find the rate the canoe is paddled, using x as this rate.
19. A loaded moving truck is traveling 30 mph faster than a freight train. In the time it takes the train to travel 135 miles, the truck travels 225 miles. Find the speed of the truck.
20. To estimate the number of fish in a lake, a park ranger catches 220 fish, tags them, and returns them to the lake. Later, 72 fish are caught, and it is found that 20 of them are tagged. Estimate the number of fish in the lake.
21. Source of Heat A will melt a pound of ice in 2 hours and Source of Heat B will melt a pound of ice in 5 hours. How long will it take to melt a pound of ice if both sources are used at the same time?

## Teaching Tips for Unit 7

1. Students need to be reminded that factoring is the key skill in many of these topics:
a. Determining values where a rational expression is undefined.
b. Reducing rational expressions to lowest terms.
c. Finding LCD.
2. Parallels should be drawn between the algebra of rational expressions and the arithmetic of rational numbers.
3. Use long division of multi-digit numbers to remind students of things frequently forgotten in polynomial long-division:
a. Divide leading terms to get terms in the quotient.
b. Subtract intermediate answers.
c. Identify quotient, dividend, and remainder.
d. Express remainder as a fraction.
4. Emphasize that the ONLY things that can be cancelled in reducing a rational expression are FACTORS: addends cannot be cancelled!
5. Checking answers is REQUIRED: students will frequently skip that step, reporting extraneous roots.
6. Most students find dividing by a binomial very confusing at first.
7. Most students understand the steps for dividing by a binomial better if a numerical long division problem is shown in parallel.
8. Show that long division can be used to find the quadratic factor of $\left(a^{3}-b^{3}\right)$ when only the linear factor $(a-b)$ is remembered.
9. Some comments can be made when doing "additive work" problems about their realism:
a. Most work in real life is not additive.
b. Extreme values will show a failure of the basic formula (if one painter can do a job in 4 hours, 250 painters could not do the job in less than 1 minute).
c. Rates of work are frequently not constant.
10. Other examples of proportions:
a. Scale models.
b. Maps.
c. Photographs.
11. Use function notation whenever possible.
12. Include geometry applications whenever possible.

## Unit 8: Rational Exponents and Radicals

## Unit Description

The student will simplify radical expressions, and use rational exponents. The student will solve radical equations and use them to solve application problems.

## Broad Learning Outcomes

Upon completion of Unit 8 students will be able to:
8.1 Demonstrate the equivalence of radical and rational exponent forms.
8.2 Compute and estimate radicals.
8.3 Simplify radicals and radical expressions.
8.4 Perform operations (add, subtract, multiply) on radicals and radical expressions.
8.5 Rationalize the denominator (one term and two terms).
8.6 Solve radical equations.
8.7 Define the imaginary unit and imaginary numbers.
8.8 Simplify square roots of negative numbers using the imaginary unit.
8.9 Solve application problems involving radicals.


### 8.7 Define the imaginary unit and imaginary numbers.

8.7.1 Define $i=\sqrt{-1}$.
8.7.2 Define imaginary numbers (e.g. $\sqrt{-25}$ ).
8.8 Simplify square roots of negative numbers using the imaginary unit.
8.9 Solve application problems involving radicals.
8.9.1 Solve problems involving right triangles.
8.9.2 Solve problems involving the Pythagorean Theorem.
8.9.3 Solve problems involving the distance formula.
$1 / 2$ academic hour
$1 / 2$ academic hour 2½ academic hours

## Sample Assessment for Unit 8

8.1.1
8.1.2
8.1.3
8.2.1
8.2.2
8.2.3
8.3.1
8.3.1
8.3.2
8.3.2
8.3.3
8.3.3
8.3.4
8.3.5

1. Rewrite the following using radical notation. Simplify, if possible. $(16)^{\frac{1}{2}}$
2. Rewrite the following using radical notation. Simplify, if possible. $(-125)^{\frac{1}{3}}$
3. Rewrite the following using radical notation. Simplify, if possible. $(9 x+2)^{\frac{2}{5}}$
4. Use the calculator to approximate the radical $\sqrt{38}$ to three decimal places.
5. Fill in the blanks. The square root of 73 is between the consecutive integers $\qquad$ and
$\qquad$ since 73 is between $\qquad$ and $\qquad$ _-.
6. Use the calculator to approximate the radical $\sqrt[3]{103}$ to three decimal places.
7. Use the properties of exponents to simplify each expression.

Write answers with positive exponents. $\frac{a^{\frac{7}{8}}}{a^{\frac{3}{8}}}$
8. Use the properties of exponents to simplify each expression.

Write answers with positive exponents. $\quad\left(x^{-\frac{4}{3}} y^{\frac{3}{5}}\right)^{15}$
9. Simplify. Assume that all variables represent any real numbers (positive or negative).
$7 \sqrt{45}$
10. Simplify. Assume that all variables represent any real numbers (positive or negative). $\sqrt{16 x^{2}}$
11. Simplify. Assume that all variables represent any real numbers (positive or negative). $\sqrt[5]{-m^{18} p^{20}}$
12. Simplify. Assume that all variables represent any real numbers (positive or negative). $\sqrt[3]{56 x^{5}}$
13. Simplify. Assume that all variables represent any real numbers (positive or negative). $\sqrt{5} \cdot \sqrt{10}$
14. Simplify. Assume that all variables represent any real numbers (positive or negative). $\frac{\sqrt{66}}{\sqrt{3}}$
8.3.5
8.5.1
8.6
8.7.1
8.7.2
15. Simplify. Assume that all variables represent any real numbers (positive or negative).
$-\sqrt{\frac{17}{64}}$
16. Which of the following are like radicals? Circle your answers.
$\sqrt{72}, 5 \sqrt{2}, \sqrt{28}, \sqrt{50}, \sqrt{12}$
17. Perform the indicated operation. Simplify if possible. $4 \sqrt{20}+\sqrt{49}-\sqrt{180}$
18.

Perform the indicated operation. Simplify if possible. $\frac{\sqrt[3]{7 x}}{12}-\sqrt[3]{\frac{7 x}{27}}$
19. Perform the indicated operation. Simplify if possible. $\sqrt{3}(\sqrt{3}-\sqrt{6 x})$
20. Perform the indicated operation. Simplify if possible. $(4-\sqrt{7})(1-2 \sqrt{7})$
21. Rationalize the denominator. $\frac{-14}{\sqrt{8 x}}$
22. Rationalize the denominator. $\frac{10}{\sqrt[3]{2}}$
23. Rationalize the denominator. $\frac{1+\sqrt{6}}{3-\sqrt{6}}$
24. Solve. $4+\sqrt[3]{x-7}=0$
25. Solve. $\sqrt{3 x-2}=x-4$
26. What is the mathematical symbol for $\sqrt{-1}$ ?
27. Which of the following represent imaginary numbers?

$$
\begin{array}{lllllll}
\sqrt{-9} & 7 i & -\sqrt{5} & (-3)^{\frac{1}{2}} & 4^{-\frac{1}{2}} & \sqrt[3]{-64} & i \sqrt{2}
\end{array}
$$

28. Simplify each of the following using the imaginary unit.
29. Simplify using the imaginary unit. $\frac{-6 \pm \sqrt{-32}}{2}$
30. A wire is needed to support a vertical pole 18 feet high. The cable is to be anchored 8 feet from the base of the pole. How much cable is needed? Approximate the answer to three decimal places.

| 8.9.2 | 31. Find the missing side. Give the exact answer as a simplified radical. |
| :--- | :--- |


32. Find the distance between the points $(9,4)$ and $(-9,10)$. Give the exact answer as a simplified radical.

## Teaching Tips for Unit 8

1. Emphasize the difference between estimating and simplifying radicals
2. Make sure students can estimate with a calculator, not only radicals, but expressions with radicals (such as $\frac{1+\sqrt{3}}{2}$ )
3. When simplifying with the imaginary unit, make sure students understand this is not a real number.
4. Use function notation whenever possible.

## Unit 9 - Functions, Quadratic Equations, and Parabolas

## Unit Description

In this unit the student will have an introduction to functions in ordered pair, graph, and equation form. The student will engage in a thorough introduction to quadratic functions and their properties as they complete preparation for entering STEM or business-administration college-level mathematics courses.

## Broad Learning Outcomes

Upon completion of Unit 9 students will be able to:
9.1 Determine if a relation is a function and identify the domain and range of the function.
9.2 Find all roots of quadratic equations using both the square root method and the quadratic formula.
9.3 Analyze a quadratic function to determine its vertex by completing the square and using the formula.
9.4 Graph a quadratic function, using the vertex form, indicating the intercepts and vertex.
9.5 Apply knowledge of quadratic functions to solve application problems from geometry, economics, applied physics, and other disciplines.

## Specific Objectives

Upon completion of Unit 9 students will be able to:
9.1 Determine if a relation is a function and identify the domain and range of the function.
9.1.1 Determine if a list of ordered pairs, graph, or equation is a function.
9.1.2 Determine the domain and range of a function given as a list of ordered pairs.
9.1.3 Determine the domain and range of a function given as a graph.
9.1.4 Determine the domain of a function given as an equation.
9.1.5 Evaluate $y=f(x)$ for constant values of $x$ and for specific monomials and binomials.
9.2 Find all roots of quadratic equations using both the square root method and the quadratic formula.
9.2.1 Find the roots of quadratic equations of the form $a x^{2}+c=0$.
9.2.2 Find the roots of quadratic equations of the form
$a x^{2}+b x+c=0$ when the discriminant is a positive perfect square, (i.e. the quadratic is factorable).
9.2.3 Find the roots of quadratic equations of the form
$a x^{2}+b x+c=0$ when the discriminant is positive, but not a perfect square.
9.2.4 Find the roots of quadratic equations of the form $a x^{2}+b x+c=0$ when the discriminant is zero.
9.2.5 Find the roots of quadratic equations of the form $a x^{2}+b x+c=0$ when the discriminant is negative.
9.2.6 Describe the roots of a quadratic based upon the discriminant in all cases.

Suggested time
2 academic hours

4 academic hours
9.3 Analyze a quadratic function to determine its vertex by completing the square and using the formula.
9.3.1 Write a quadratic function in vertex form $y=a(x-h)^{2}+k$ by completing the square for quadratics with $a=1$, and identify the vertex $(h, k)$.
9.3.2 Write a quadratic function in vertex form $y=a(x-h)^{2}+k$ by completing the square for quadratics with $a \neq 1$, and identify the vertex $(h, k)$.
9.3.3 Find the vertex of a quadratic equation $y=a x^{2}+b x+c$ using the formula method $\left(\frac{-b}{2 a}, f\left(\frac{-b}{2 a}\right)\right)$.
9.4 Graph a quadratic function, using the vertex form, indicating the intercepts and vertex.
9.4.1 Determine whether the parabola opens upward or downward.
9.4.2 Plot the vertex of the parabola.
9.4.3 Determine the axis of symmetry for the parabola.
9.4.4 Plot the $x$-intercepts of the parabola, if they exist.
9.4.5 Plot the $y$-intercept of the parabola and complete the graph with additional points as needed.
9.5 Apply knowledge of quadratic functions to solve application problems from geometry, economics, applied physics, and other disciplines.
9.5.1 Solve problems involving area optimization.
9.5.2 Solve problems involving revenue optimization.
9.5.3 Solve problems involving the motion of falling objects.

3 academic hours

3 academic hours

3 academic hours

## Sample Assessment for Unit 9

1. Of the following relations, circle those that represent functions:
(a) $x=y^{2}$
(b) $y=x^{2}$
(c) $\{(1,1),(1,2)\}$
(d) $\{(1,1),(2,1)\}$
(e) $x=|y|$
2. Of the following relations, circle those that represent functions:
a.

d.

b.

c.

e.

3. State the domain of the function defined by the set of ordered pairs.
$\{(-3,2),(-1,0),(4,5),(7,-8)\}$
4. State the range of the function defined by the set of ordered pairs.
$\{(-3,2),(-1,0),(4,5),(7,-8)\}$
5. State the domain and range of the function defined by the graph.

6. State the domain of the function $f(x)=\sqrt{x-1}$
7. State the domain of the function $f(x)=\frac{x-1}{x+2}$
8. Given $f(x)=3-2 x-x^{2}$ evaluate $f(-1)$.
9. For the function $f(x)=x^{2}-1$ evaluate $f(a)$.
10. For the function $f(x)=3 x^{2}-2 x+4$ evaluate $f(x+h)$.
11. Find the roots of $2 x^{2}-4=0$.
12. Find the roots of $3 x^{2}+x-2=0$.
13. Find the roots of $x^{2}-x-1=0$.
14. Find the roots of $x^{2}-4 x+4=0$.
15. Find the roots of $x^{2}+4 x+6=0$.
16. Describe the roots of a quadratic function by matching the following where $D=b^{2}-4 a c$ is the discriminant of the quadratic formula:
(a) $D>0$ and is not a perfect square
(b) $D$ is a perfect square
(c) $D=0$
(d) $D<0$
$\qquad$ (I.) The quadratic has no real roots.
$\qquad$ (II.) The vertex of the quadratic is on the x -axis.
$\qquad$ (III.) The quadratic can be factored.
$\qquad$ (IV.) The quadratic has two distinct irrational roots.
17. Write $f(x)=x^{2}+6 x+6$ in vertex form and indicate the coordinates of the vertex.
18. Write $f(x)=2 x^{2}+4 x+1$ in vertex form and indicate the coordinates of the vertex.
19. Identify the vertex of $f(x)=2 x^{2}+12 x+17$ by using the formula method. For questions 20 through 26 you are given $f(x)=x^{2}-6 x+8$.
20. Does the graph representing this equation open upward or downward? How do you know?
21. Determine the vertex of the parabola by putting the equation in vertex form.
22. Determine the axis of symmetry of the parabola.
23. Determine the $x$-intercepts of the parabola.
24. Determine the $y$-intercept of the parabola.
25. Determine the points on the parabola when $x=1$ and when $x=-1$.
26. On the grid given, make a sketch of the parabola. Be sure to indicate the vertex, axis of symmetry, $x$-intercepts, and y-intercept. Label the graph appropriately.

9.5.1
27. A rectangular-shaped vegetable garden next to a barn is to be fenced on three sides with 120 total feet of fencing. Find the dimensions of the garden that will maximize the area.

28. If the revenue function for a market gardener growing a crop of tomatoes is given by $R(x)=$ $-x^{2}+50 x$ where $x$ is the number of bushels grown, what is the maximum revenue possible? How many bushels are sold to yield this maximum revenue?
29. A flare is fired directly upward into the air from a boat that is experiencing engine problems. The height of the flare (in feet) above the water, $t$ seconds after being fired is given by the model $h(t)=-16 t^{2}+96 t+5$. If the flare is designed to explode when it reaches its highest point, at what height will this occur? How many seconds after the flare is fired does it explode?
30. Can a function have two y-intercepts? Explain your answer.
31. Is it possible for the range of a quadratic function to equal all real numbers? Explain your answer.

## Teaching Tips for Unit 9

1. When introducing the concept of function, one way we can motivate the definition is by thinking of making a model for the position of a moving object as a function of time. The object occupies exactly one position at a particular time. You might also have fun with the class and ask each student to map themselves to their birth month.
2. As you work with identifying functions, emphasize the vertical line test where possible and show how it can be used not just with graphs but with ordered pairs also.
3. To determine domain, have students imagine that the graph of a function is coated in bright red paint. Now mentally slide the graph up and down through the $x$-axis. Where does the $x$-axis have paint on it? That's your domain. Then translate that information into interval notation. Follow a similar procedure for range except slide the graph left and right through the $y$-axis.
4. For factorable quadratics, show how the quadratic formula "tells" you in that case (because the formula yields rational roots) that you could factor instead.
5. You can start early with the concept of completing the square as you derive the quadratic formula. To motivate completing the square, you can show what a perfect square binomial looks like and work backwards, identifying the constant as half the linear coefficient squared.
6. Remind students to "work smarter not harder" by factoring when possible in lieu of using the quadratic formula.
7. Connect the vertex formula and completing the square by completing the square on $y=a x^{2}+$ $b x+c$.
8. It is best to pick one or two general quality examples that cover a particular concept, rather than many different examples. This will lessen the required lecture time and allow students more time to be engaged in completing problems themselves.

## Appendix A

## Unit 0 - Operations with Whole Numbers

## Unit Description

The student will solve contextual problems using whole numbers. All student learning outcomes for this unit must be completed without the use of a calculator. Emphasis should be placed on applications throughout the unit.

## Broad Learning Outcomes

Upon completion of Unit 0 students will be able to:
0.1 Recognize place value and names for numbers
0.2 Perform operations with whole numbers
0.3 Round whole numbers and estimation with whole numbers
0.4 Solve application problems by adding, subtracting, multiplying, or dividing whole numbers

## Specific Objectives

Upon completion of Unit 0 students will be able to:
0.1 Recognize place value and names for numbers
0.1.1 Find the place value of a digit in a whole number
0.1.2 Write a whole number in words
0.1.3 Write a whole number in standard form
0.2 Perform operations with whole numbers
0.2.1 Add whole numbers
0.2.2 Find the perimeter of a polygon
0.2.3 Subtract whole numbers including borrowing when necessary
0.2.4 Multiply whole numbers
0.2.5 Find the area of a rectangle
0.2.6 Divide whole numbers
0.2.7 Find the average of a list of numbers
0.2.8 Use order of operations to simplify whole number expressions with multiple operations
0.3 Round whole numbers and estimate sums, differences, products and quotients with whole numbers
0.3.1 Round whole numbers to a specified place value
0.3.2 Use rounding to estimate sums, differences, products and quotients with whole numbers
0.4 Solve application problems by adding, subtracting, multiplying, or dividing whole numbers

## Appendix B

## Sample Syllabi

## Unit 2 : Operations with Positive Decimals and Percents

| Professor: |  |
| :---: | :---: |
| Contact Information |  |
| Office Hours |  |
| Textbooks and Supplies | (include ISBNs) |
| Placement | Placement in this course is based upon the college's placement test or successful completion of the Unit 1. |
| Unit <br> Description | The student will solve problems using decimals and percents. Emphasis should be placed on applications throughout the unit. Applications will use U.S. customary and metric units of measurement. |
| General Objectives (list the broad learning outcomes for the unit) | Upon completion of Unit 2 students will be able to: <br> - Demonstrate the meaning of decimal numbers. <br> - Perform operations with decimals. <br> - Estimate decimals. <br> - Demonstrate the relationship among fractions, decimals, and percents. <br> - Solve basic percent problems. <br> - Read and interpret basic graphs. <br> - Convert units of measure. <br> - Solve application problems using U.S. customary and metric units of measurement. Specific objectives are below in the class schedule. |
| Grading |  |
| Attendance |  |
| Makeups |  |
| Homework |  |
| Quizzes and Tests |  |
| Evacuations and Inclement weather | For example: Closings and delays will be posted on the $\qquad$ web site www. $\qquad$ .edu and announced on radio and television. You may also sign up for e-alert through the $\qquad$ website to receive a text message on your cell phone. (This method is highly |



Effective Date:

## Sample Syllabus

## Unit 8 - Rational Exponents and Radicals

1. Instructor:

Office Hours:

Office Telephone Number:
Email:
Office Location:
2. Prerequisites for this course are Developmental Math Unit 7, or qualifying score on the math placement test.
3. Unit Description and Learning Outcomes::

The student will simplify radical expressions, and use rational exponents. The student will solve radical equations and use them to solve application problems.

Upon completion of Unit 8 students will be able to:

- Demonstrate the equivalence of radical and rational exponent forms.
- Compute and estimate radicals.
- Simplify radicals and radical expressions.
- Perform operations (add, subtract, multiply) on radicals and radical expressions.
- Rationalize the denominator (one term and two terms).
- Solve radical equations.
- Define the imaginary unit and imaginary numbers.
- Simplify square roots of negative numbers using the imaginary unit.
- Solve application problems involving radicals.

4. Textbook and other materials and supplies (Include ISBNs)
5. Learning Activities: (list the types of activities planned to cover material such as lecture format, assignments, etc.)
6. Applicable Learning Resources: the textbook, class notes, handouts, (list other resources such as videos, CD's, tutoring, etc).
7. Evaluation:

List here how the final course grade will be calculated including tests, quizzes, homework, projects, final exams, etc. Provide the grading scale used for determining the final course grade.
8. Midterm Assessment: A midterm report is to be given to each student before the last day to withdraw from the course using the departmental form or a personally designed form.
9. Attendance Requirements: Example: The College Attendance Policy states

Attendance will be kept by circulating an attendance sheet for each class meeting. Failure to sign the sheet will result in a recorded absence for that class. Instructors should also inform students of any additional attendance requirements for the courses they teach.
10. Students with Disabilities
11. Student learning outcomes may be attached, or may be obtained at the School/Division Office.
12. Schedule of Class Events:

The instructor should provide the chapters, sections, time frame (if known), and order of coverage intended to be used throughout the course. If specific dates for tests are known, they can be listed on the syllabus.
A sample schedule for a three-week course is as follows:
Week 1:

- Demonstrate the equivalence of radical and rational exponent forms.
- Compute and estimate radicals.
- Simplify radicals and radical expressions.


## Quiz/Assessment

Week 2:

- Perform operations (add, subtract, multiply) on radicals and radical expressions. - Rationalize the denominator (one term and two terms).


## Quiz/Assessment

Week 3:

- Solve radical equations.
- Define the imaginary unit and imaginary numbers.
- Simplify square roots of negative numbers using the imaginary unit.
- Solve application problems involving radicals.

Final Test/Exam

